

Spin-gap phase in the extended $t - J$ chain

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Abstract. We study the one-dimensional deformed $t - t' - J$ model in terms of the continuum field theories. We found that at low doping concentration and far away from the phase separation regime, there are two phases: the Luttinger liquid and the Luther-Emery liquid, depending on $t'/t < (t'/t)_c$ or $t'/t > (t'/t)_c$, where $(t'/t)_c > 0$. Moreover, the singlet superconducting correlations are dominant in the Luther-Emery liquid.

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1 Introduction

Strong electronic correlations are widely believed to be crucial for the understanding of the anomalous properties of high-temperature superconductors. A popular approach in this context is the use of the $t - J$ model, with holes moving in an antiferromagnetic (AF) spin background. (Here t and J are the nearest-neighbor hopping amplitude and the AF exchange interaction, respectively.) In recent years, the measurements of angle resolved photoemission spectroscopy (ARPES) in $\text{Sr}_2\text{CuO}_2\text{Cl}_2$ [1] indicate that it is necessary to include a next-nearest-neighbor hopping term to explain the ARPES data, which leads to the “extended” $t - J$ model. Subsequent efforts have concentrated on the effects of the extra hopping terms on various properties of planar and ladder systems, such as stripe stability [2], competition between stripes and pairing [3], spin-charge separation in two dimensions [4], and spin gap evolution in two-leg ladders [5]. Currently, it is well-established that a positive value of the next-nearest-neighbor hopping t' enhances hole pairing, while the opposite occurs for t' negative [3].

In the present paper, we study the deformed version of the one-dimensional (1D) $t - t' - J$ model (see the following). Our main results are as follows: (i) Far away from the phase separation regime ($J/t \ll 1$), there are two phases at low doping concentration: the Luttinger liquid (LL) for $t'/t < (t'/t)_c$ and the Luther-Emery (LE) liquid (spin-gap phase) for $t'/t > (t'/t)_c$, where $(t'/t)_c > 0$. (The LE liquid is a spin-gapped state with one gapless charge mode.) (ii) The value of $(t'/t)_c$ is expected to depend on the hole concentration x and the ratio J/t . At low hole concentration ($x \ll 1$), however, it is insensitive to the value of J/t and

is an increasing function of x . (iii) The spin gap is opened in the LE liquid and thus both $2k_F$ charge density wave (CDW) and singlet superconducting (SS) correlations are enhanced. But it is the SS one which is dominant in the LE liquid.

The rest of this paper is organized as follows: The deformed $t - t' - J$ model and the corresponding continuum theory are introduced in Section 2. Section 3 is devoted to the phase diagram of the 1D $t - t' - J$ model. We study the properties of the ground state in the spin-gap phase in Section 4. The last section is the conclusions and discussions of our results.

2 Deformed $t - t' - J$ model

We start with the 1D $t - t' - J$ model which is described by the Hamiltonian

$$H = H_h + H_J,$$

where

$$H_h = -t \sum_j \left(\bar{c}_{j+1\alpha}^\dagger \bar{c}_{j\alpha} + \text{h.c.} \right) - t' \sum_j \left(\bar{c}_{j+2\alpha}^\dagger \bar{c}_{j\alpha} + \text{h.c.} \right), \quad (1)$$

$$H_J = J \sum_j \left(\mathbf{S}_j \cdot \mathbf{S}_{j+1} - \frac{1}{4} n_j n_{j+1} \right). \quad (2)$$

Here the Hubbard operator [6] $\bar{c}_{j\alpha}$ is given by

$$\bar{c}_{j\alpha}^\dagger = c_{j\alpha}^\dagger (1 - n_{j-\alpha}), \quad (3)$$

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where $c_{j\alpha}$ is the annihilation operator of electrons on the site j with spin α , and $\alpha = \uparrow, \downarrow$ correspond to spin up and down, respectively. Moreover, $n_j = c_{j\alpha}^\dagger c_{j\alpha}$ and $\mathbf{S}_j = \frac{1}{2} c_{j\alpha}^\dagger (\boldsymbol{\sigma})_{\alpha\beta} c_{j\beta}$. In the following, we shall take $t, J > 0$. The extra factor in the Hubbard operator is to impose the no-double-occupancy condition which results from the strong Coulomb repulsion between two electrons with opposite spins on the same site. Inserting equation (3) into H_h leads to four- and six-fermion interactions with the same strengths as the hopping amplitudes, which defies the perturbative approach.

To overcome this difficulty, Chen and Wu proposed to replace the Hubbard operator (3) by the deformed one [7]

$$\tilde{c}_{j\alpha}^\dagger = c_{j\alpha}^\dagger (1 - \Delta n_{j-\alpha}), \quad (4)$$

with the deformation parameter $0 < \Delta \leq 1$. That is, a non-zero probability to leak into the states with double occupancy is now allowed. The deformed model has the advantage that as $0 < \Delta \ll 1$ and $J/t \ll 1$, all four- and six-fermion interactions can be treated as perturbations and thus a field-theoretical approach can be performed. It is hoped that both the $t - t' - J$ model ($\Delta = 1$) and the deformed one ($0 < \Delta \ll 1$) fall into the same universality class.

Now we would like to study the low-energy physics of the 1D deformed $t - t' - J$ model away from the phase separation regime. This can be achieved by studying the corresponding continuum field theory. Inserting equation (4) into equations (1) and (2), expanding the electron operator around the Fermi points by

$$c_{j\alpha} \approx \sqrt{a_0} [e^{-ik_F x} \psi_{L\alpha}(x) + e^{ik_F x} \psi_{R\alpha}(x)], \quad (5)$$

where $x = ja_0$ with a_0 being the lattice spacing and k_F is the Fermi momentum determined by the electron density n via $k_F a_0 = \frac{\pi}{2} n$, ($n = 1$ corresponding to the half-filling.) and then taking the continuum limit ($a_0 \rightarrow 0$) by keeping ta_0 , $t'a_0$, and Ja_0 finite, we found that the Hamiltonian describing the dynamics of ψ -fermions, up to the four-fermion interactions, is given by

$$H_\psi = \int dx (H_c + H_s),$$

where

$$H_c = \frac{\pi}{2} v_{c0} : (J_L J_L + J_R J_R) : + \lambda_c J_L J_R, \quad (6)$$

$$H_s = \frac{2\pi}{3} v_s : (\mathbf{J}_L \cdot \mathbf{J}_L + \mathbf{J}_R \cdot \mathbf{J}_R) : + \lambda_s \mathbf{J}_L \cdot \mathbf{J}_R, \quad (7)$$

with

$$\begin{aligned} J_{L(R)} &= : \psi_{L(R)\alpha}^\dagger \psi_{L(R)\alpha} : , \\ \mathbf{J}_{L(R)} &= \frac{1}{2} : \psi_{L(R)\alpha}^\dagger (\boldsymbol{\sigma})_{\alpha\beta} \psi_{L(R)\beta} : . \end{aligned} \quad (8)$$

In the above, $: \cdots :$ stands for normal ordering with respect to the Fermi points. The parameters appearing in

equations (6) and (7) are defined by $v_{c0} = v_F [1 + 2g_c / (\pi v_F)]$, $v_s = v_F [1 + 3g_s / (2\pi v_F)]$, and

$$\begin{aligned} \lambda_c &= 4 \left\{ \Delta \left(1 - \Delta \frac{n}{2} \right) \left[t \cos \left(\frac{\pi n}{2} \right) + 2t' \cos(\pi n) \right] \right. \\ &+ \frac{\Delta^2}{\pi} \sin \left(\frac{\pi n}{2} \right) \left[t \cos(\pi n) + 4t' \cos \left(\frac{\pi n}{2} \right) \cos(2\pi n) \right] \\ &\left. - \frac{J}{4} \cos^2 \left(\frac{\pi n}{2} \right) \right\}, \end{aligned} \quad (9)$$

$$\begin{aligned} \lambda_s &= -16 \left\{ \Delta \left(1 - \Delta \frac{n}{2} \right) \left[t \cos \left(\frac{\pi n}{2} \right) + 2t' \cos(\pi n) \right] \right. \\ &+ \frac{\Delta^2}{\pi} \sin \left(\frac{\pi n}{2} \right) \left[t + 4t' \cos \left(\frac{\pi n}{2} \right) \right] \\ &\left. - \frac{J}{4} \cos^2 \left(\frac{\pi n}{2} \right) \right\}, \end{aligned} \quad (10)$$

where $v_F = 2t(1 - \Delta \frac{n}{2})^2 \sin k_F \times [1 + 4\frac{t'}{t} \cos k_F]$ is the Fermi velocity, and

$$\begin{aligned} g_c &= 2\Delta \left(1 - \Delta \frac{n}{2} \right) \left[t \cos \left(\frac{\pi n}{2} \right) + 2t' \cos(\pi n) \right] \\ &+ \left[t \cos(\pi n) + 4t' \cos \left(\frac{\pi n}{2} \right) \cos(2\pi n) \right] \\ &\times \frac{\Delta^2}{\pi} \sin \left(\frac{\pi n}{2} \right) - \frac{J}{4}, \\ g_s &= -\frac{4}{3} \left\{ 2\Delta \left(1 - \Delta \frac{n}{2} \right) \left[t \cos \left(\frac{\pi n}{2} \right) + 2t' \cos(\pi n) \right] \right. \\ &+ \left[t(2 + \cos(\pi n)) + 4t' \cos \left(\frac{\pi n}{2} \right) (2 + \cos(2\pi n)) \right] \\ &\left. \times \frac{\Delta^2}{\pi} \sin \left(\frac{\pi n}{2} \right) - \frac{3}{4} J \right\}. \end{aligned}$$

We have set $a_0 = 1$ in the above expressions. In equations (9) and (10), the terms proportional to $\Delta^2 \sin(\pi n/2)$ arise from the six-fermion interactions in H_h , whereas the four-fermion interactions in H_h contribute to those proportional to $\Delta(1 - \Delta n/2)$. (For the details of derivation, see Appendix A.) Note that the above equations are derived by assuming $n \neq 1$ and thus the Umklapp processes are irrelevant operators which can be neglected. The higher order interactions between ψ -fermions are neglected in H_ψ . This is because in the weak-coupling regime ($\Delta \ll 1$) they are irrelevant operators in the sense of renormalization group.

Before exploring the physics contained by equations (6, 7, 9), and (10), three points have to be mentioned. First, the Umklapp processes are neglected, which changes the low-energy physics of the charge sector at half-filling drastically. Therefore, the following analysis about the charge sector cannot be extrapolated to that case. Next, the inclusion of t' modifies the dispersion relation of free electrons. It is possible that there are four Fermi points instead of two upon hole or electron doping. The low-energy physics in this case will be similar to that of the two-band

system. However, this will occur only when $|t'/t| > 0.25$ and the doping concentration exceeds some critical value. On the other hand, the above derivation starts with the one-band assumption — equation (5). Thus, we shall apply equations (6, 7, 9), and (10) to the following situations: (i) finite doping concentrations for $|t'/t| < 0.25$ and (ii) light doping for $|t'/t| > 0.25$. Finally, equations (9) and (10) are valid only when $\Delta \ll 1$ and $J/t \ll 1$. Since the 1D $t - J$ model is phase separated as $J/t \geq O(1)$ [8], the Hamiltonian H_ψ describes the low-energy physics in the region far away from the place where the phase separation occurs.

3 Phase diagram

Because of the spin-charge separation, the dynamics of the charge and spin sectors can be discussed separately. We first consider the charge sector. In terms of the bosonization formulas,

$$\begin{aligned} J_L + J_R &= \sqrt{\frac{2}{\pi}} \partial_x \Phi_c, \\ J_L - J_R &= \sqrt{\frac{2}{\pi}} \partial_x \Theta_c, \end{aligned} \quad (11)$$

the Hamiltonian H_c can be written as

$$H_c = \frac{v_c}{2} : \left[K_c (\partial_x \Theta_c)^2 + \frac{1}{K_c} (\partial_x \Phi_c)^2 \right] :, \quad (12)$$

where $v_c = v_{c0} \sqrt{1 - [\lambda_c / (\pi v_{c0})]^2}$ and

$$K_c = \sqrt{\frac{1 - \lambda_c / (\pi v_{c0})}{1 + \lambda_c / (\pi v_{c0})}}. \quad (13)$$

Accordingly, the charge sector is gapless away from half-filling.

Next we turn into the spin sector. The relevancy of the coupling λ_s in H_s can be determined by the one-loop RG equation, which is given by

$$\frac{d\lambda_s(l)}{dl} = \frac{1}{2\pi} \lambda_s^2(l). \quad (14)$$

For simplicity, we have set $v_s = 1$ in equation (14). From equation (14), we see that for $\lambda_s(0) < 0$, λ_s flows to zero and the spin excitations are gapless. On the other hand, $\lambda_s(l \rightarrow \infty) \rightarrow \infty$ as $\lambda_s(0) > 0$. In the latter case, the low energy physics can be elucidated by abelian bosonization. Using the bosonization formulas

$$\begin{aligned} J_L^+ J_R^- + J_L^- J_R^+ &= -\frac{1}{2\pi^2 a_0^2} \cos(\sqrt{8\pi} \Phi_s), \\ J_L^z + J_R^z &= \frac{1}{\sqrt{2\pi}} \partial_x \Phi_s, \\ J_L^z - J_R^z &= \frac{1}{\sqrt{2\pi}} \partial_x \Theta_s, \end{aligned} \quad (15)$$

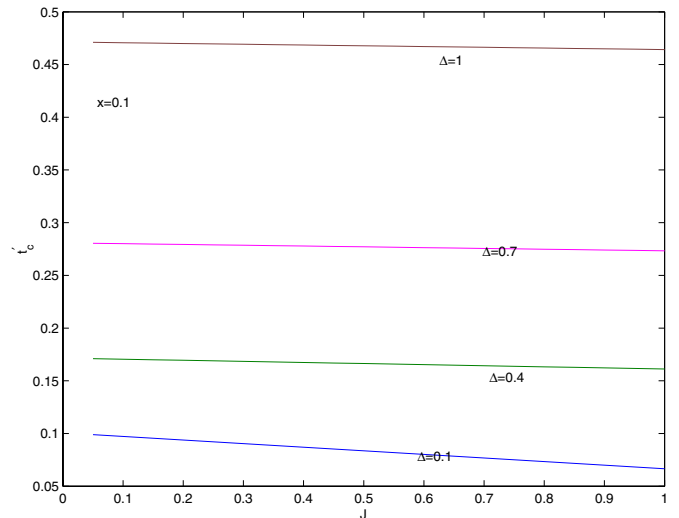


Fig. 1. The critical value of t'_c at $x = 0.1$ and $t = 1$ for different values of the deformed parameter Δ .

where $J_{L(R)}^\pm = J_{L(R)}^x \pm iJ_{L(R)}^y$, the λ_s term becomes

$$\begin{aligned} \lambda_s \mathbf{J}_L \cdot \mathbf{J}_R &= \lambda_s \left\{ \frac{1}{8\pi} [(\partial_x \Phi_s)^2 - (\partial_x \Theta_s)^2] \right. \\ &\quad \left. - \frac{1}{4\pi^2 a_0^2} \cos(\sqrt{8\pi} \Phi_s) \right\}. \end{aligned}$$

When λ_s is positive, the cosine term becomes relevant and $\langle \Phi_s \rangle = \sqrt{\pi/2} l$ where l is some integer. This results in a spin gap.

To proceed, we have to use the expression of λ_s (10). For given $x > 0$, $J/t \ll 1$, and Δ , we found that λ_s will change its sign from $\lambda_s < 0$ to $\lambda_s > 0$ by increasing the value of t'/t , where $x = 1 - n$ is the hole concentration. Although equation (10) is derived under the assumption $\Delta \ll 1$, we shall extrapolate it to $\Delta = 1$ to get a rough estimate of the phase boundary. The critical value $(t'/t)_c$ is determined by the equation $\lambda_s = 0$. The solution of it gives the dependence of $(t'/t)_c$ on x and J/t , and the results are depicted in Figures 1 and 2. As mentioned at the end of Section 2, the proper starting point to treat the 1D $t - t' - J$ model is the two-band model instead of the one-band model we employed in this paper for $|t'/t| > 0.25$ and moderate values of x . Therefore, we only consider the case with low doping concentrations in both figures. Figure 1 gives $(t'/t)_c$ as a function of J/t at $x = 0.1$ for $\Delta = 0.1, 0.4, 0.7, 1$. For fixed x and Δ , $(t'/t)_c$ is a decreasing function of J/t . But in the range we considered it is, in fact, insensitive to the value of J/t at low doping concentration except for small values of Δ . On the other hand, for fixed J/t and Δ , the hole concentration x has dramatic effects on $(t'/t)_c$. This can be seen in Figure 2, which shows that $(t'/t)_c$ is an increasing function of x .

The above study indicates that the low-energy physics of the deformed 1D $t - t' - J$ model is described by the LLs for $t'/t < (t'/t)_c$, and it becomes a LE liquid as $t'/t > (t'/t)_c$. In addition, Figures 1 and 2 suggest that

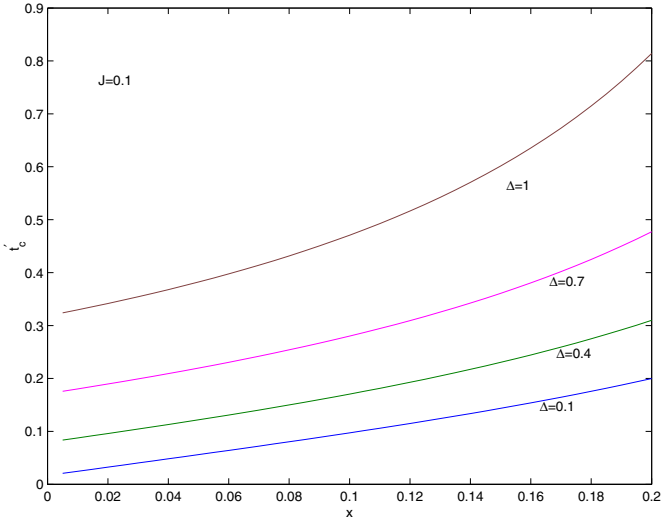


Fig. 2. The critical value of t'_c at $J = 0.1$ and $t = 1$ for different values of the deformed parameter Δ .

$(t'/t)_c > 0$ even for $\Delta = 1$. In fact, by extrapolating equation (10) into $\Delta = 1$, we found $(t'/t)_c \approx 0.32$ as $x \rightarrow 0^+$. (Note that the 1D $t - J$ model falls into the LL phase [9].) Accordingly, a positive t' favors the opening of a spin gap and thus suppresses the AF ordering. Moreover, the J term plays a minor role in determining the phase diagram at lightly doping as long as the system is far away from the region of phase separation. This can be seen by expanding equation (10) according to the power of x . Then, the dependence of λ_s on J/t starts from $O(x^2)$, whereas its dependence on t'/t starts from $O(1)$.

4 Spin-gap phase

To further understand the properties of the ground state in the spin-gap phase, we first examine the $2k_F$ CDW and SS susceptibilities. The corresponding order parameters are defined by

$$\begin{aligned} O_{CDW} &= \psi_{L\alpha}^\dagger \psi_{R\alpha}, \\ O_{SS} &= \frac{i}{2} \epsilon_{\alpha\beta} \psi_{L\alpha} \psi_{R\beta}. \end{aligned} \quad (16)$$

In terms of the bosonization formulas

$$\psi_{L(R)\alpha} = \frac{1}{\sqrt{2\pi a_0}} \eta_\alpha \exp \left\{ \mp i\sqrt{4\pi} \phi_{L(R)\alpha} \right\}, \quad (17)$$

where $\eta_{\uparrow(\downarrow)}$ are real fermions which satisfy $\eta_{\uparrow(\downarrow)}^2 = 1$, they can be bosonized as

$$\begin{aligned} O_{CDW} &= \frac{i\gamma}{\pi a_0} \exp \left\{ i\sqrt{2\pi} \Phi_c \right\}, \\ O_{SS} &= \frac{i\gamma}{2\pi a_0} \eta_\uparrow \eta_\downarrow \exp \left\{ i\sqrt{2\pi} \Theta_c \right\}, \end{aligned} \quad (18)$$

where $\Phi_c = (\phi_\uparrow + \phi_\downarrow)/\sqrt{2}$, $\Phi_s = (\phi_\uparrow - \phi_\downarrow)/\sqrt{2}$, and $\gamma = \langle \cos(\sqrt{2\pi} \Phi_s) \rangle$. Using equation (18), we obtain the long

distance behaviors of the $2k_F$ CDW and SS susceptibilities

$$\begin{aligned} \langle O_{CDW}(\tau, x) O_{CDW}^\dagger(0, 0) \rangle &\sim \frac{1}{|z|^{K_c}}, \\ \langle O_{SS}(\tau, x) O_{SS}^\dagger(0, 0) \rangle &\sim \frac{1}{|z|^{K_c^{-1}}}, \end{aligned} \quad (19)$$

where $z = v_c \tau + ix$. Equation (19) indicates that the dominant fluctuations are the SS for $K_c > 1$ and the $2k_F$ CDW for $K_c < 1$. To determine which one occurs in the spin-gap phase, we resort to equation (9). In terms of λ_s , λ_c can be expressed by

$$\lambda_c = -\frac{1}{4} \lambda_s - \frac{4\Delta^2}{\pi} \cos\left(\frac{\pi}{2}x\right) f(x), \quad (20)$$

where $f(x) = t[1 + \cos(\pi x)] + 4t' \sin\left(\frac{\pi}{2}x\right)[1 - \cos(2\pi x)]$. Note that $f(x) > 0$ for $0 < x < 1$ and $t' > 0$. The spin-gap phase corresponds to $\lambda_s > 0$, which results in $\lambda_c < 0$ from equation (20). Plugging this into equation (13) gives $K_c > 1$. Although both the SS and $2k_F$ CDW correlations are enhanced in the spin-gap phase compared with the free fermions, the SS one is dominant.

To seek the origin of the enhancement of the SS correlations in the spin-gap phase, we notice that the structure of H_s (7) is identical to the continuum one for the spin-1/2 Heisenberg chain with the additional next-nearest-neighbor exchange interaction J' (the $J - J'$ model) [10]. In that case, the low energy physics is described by the LL as $J'/J < (J'/J)_c$, where $(J'/J)_c$ denotes the critical value of J'/J , because a negative value of λ_s flows to zero under the RG transformations. By increasing the value of J' such that $J'/J > (J'/J)_c$ and thus λ_s becomes positive, then $\lambda_s(l \rightarrow \infty) \rightarrow \infty$ and the spin gap is opened [10]. In the latter situation, the opening of the spin gap is associated with the occurrence of the spin-Peierls ordering, which breaks the symmetry of translation by one site (the Z_2 symmetry) spontaneously. This comparison implies that in the presence of a positive t' , the spin sector of the $t - t' - J$ model has the tendency toward the spin-Peierls ordering *without* including the J' term. (The spontaneous breaking of the Z_2 symmetry in the spin background is restored by the hole motion in the present case. See Eq. (22).) On the other hand, the lowest-energy spin excitations in the spin-gap phase are also the massive spinons, which are the kink or anti-kink connecting two degenerate ground states. This observation on the spin excitation spectrum provides another evidence to support that the underlying spin background in the spin-gap phase is the spin-Peierls state. Finally, we examine the correlations of the dimerization operator:

$$\epsilon(x) = \mathbf{S}_j \cdot \mathbf{S}_{j+1} - \mathbf{S}_{j+1} \cdot \mathbf{S}_{j+2}, \quad (21)$$

where $x = ja_0$. Then, for $|x| \rightarrow \infty$, we have

$$\langle \epsilon(x) \epsilon(0) \rangle \sim \gamma^2 \frac{\cos(2k_F x)}{x^{K_c}}. \quad (22)$$

Since $2k_F = \pi(1-x)$, at lightly doping ($x \ll 1$), the cosine term in equation (22) locally mimics the alternation that is

characteristic of a dimerized spin chain (the $J - J'$ model). The corresponding spin-Peierls ordering is weakened by a power law due to the charge fluctuations.

To sum up, the effect of a positive t' is to enhance the spin-Peierls ordering (and suppress the AF ordering) such that the whole system behaves like a doped spin-Peierls state. The doped holes in the spin-Peierls state are inclined to form the local hole pairs due to the energetic consideration. Furthermore, the calculation of the SS correlator indicates that the other effect of a positive t' is to enhance the Josephson tunneling between these local hole pairs, which results in the phase coherence between them [11].

5 Conclusions and discussions

We obtain the phase diagram of the 1D $t - t' - J$ model by studying the deformed version of it. We found that the effects of the positive t' at lightly doping are (i) to open the spin gap by enhancing the spin-Peierls ordering (and suppressing the AF ordering), and (ii) to make the hole-pair propagation become coherent and thus enhance the SS correlations. In other words, a positive t' provides a mechanism for the occurrence of the 1D superconductors in the *one-band* system with purely repulsive force.

Our findings about the effects of a positive t' are reminiscent of the role of the next-nearest-neighbor exchange interaction, J' , played on the spin-1/2 Heisenberg chain. The existence of a spin-gap phase in the 1D $t - J - J'$ model at low doping concentration was indeed predicted, as long as $J'/J > (J'/J)_c \approx 0.24$ and $J/t < (J/t)_p$, where $(J/t)_p = O(1)$ denotes the boundary of phase separation [12, 13]. The underlying reason is that a spin gap already exists at half-filling for $J'/J > (J'/J)_c$ and it is stable against the lightly hole doping. As a matter of fact, the opening of the spin gap in both the $t - t' - J$ and $t - J - J'$ models results from the same operator. From the viewpoint of the Hubbard model, i.e. the $t - t' - U$ model [14], a J' term in the low-energy effective Hamiltonian will be generated from the t' term in the large- U limit, where $U > 0$ denotes the on-site interactions between electrons. However, the value of J' generated by this way is given by $J'/J \sim (t'/t)^2$. According to our results (Fig. 2), the induced J' is still in the region $J'/J < (J'/J)_c$ at lightly doping when $t'/t > (t'/t)_c$. Therefore, it is less possible that the spin-gap phase we found arises from the J' term induced by a positive t' though the spin-gap phases in both models belong to the same universality class.

It should be mentioned that our results about the opening of the spin gap in the deformed $t - t' - J$ chain are also valid at half-filling, i.e. $n = 1$. In that case, the Umklapp processes ignored in our analysis become relevant and thus a charge gap is opened, which corresponds to $K_c = 0$. Because the Umklapp processes are composed of the spin-singlet operators, they do not affect the spin sector, especially equation (10). Therefore, there still exists $(t'/t)_c > 0$ at half-filling such that there is an algebraic long range AF order with gapless spin excitations for

$t'/t < (t'/t)_c$, whereas a spin liquid phase with the long-ranged spin-Peierls order emerges for $t'/t > (t'/t)_c$. (Note that at half-filling, equation (22) gives $\langle \epsilon(x) \rangle \sim (-1)^x \gamma$.) In this sense, the deformed $t - t' - J$ model at half-filling behaves like the $J - J'$ model instead of the Heisenberg chain (or the $t - t' - J$ chain at half-filling), which is known to be gapless in its spin sector. As mentioned in the previous paragraph, an effective J' term will be generated from the t' term in the strong coupling regime of the $t - t' - U$ model, which also contains both the Luttinger liquid and spin-gap phases. Compared with our results, it seems to suggest that in the case of half-filling the deformed $t - t' - J$ model may belong to the same universality class of the $t - t' - U$ model in the strong coupling regime, instead of the $t - t' - J$ model. Whether this discrepancy at half-filling is a generical feature or not needs more work to clarify it.

The application of our analysis on the deformed $t - J$ type model to the usual one ($\Delta = 1$) is based on the assumption that the deformed model with $\Delta \ll 1$ is adiabatically connected to the one with $\Delta = 1$. That is, there is no phase transition by increasing the deformation parameter Δ from $\Delta \ll 1$ to $\Delta = 1$. This assumption has been examined in reference [7] where the phase diagram of the $t - J_z$ model was studied. The phase diagram obtained by the deformed model is identical to that predicted by the numerical methods. This supports the use of the technique of the deformed Hubbard operator to study the $t - J$ type models. However, a recent numerical work on the $t - t' - J$ chain indicated that even moderate values of t' results in the breakdown of the LLs [15]. In addition, the low-doping phase shows similarities with the doped two-leg $t - J$ ladders where the Fermi surface takes the form of a hole pocket and the quantum numbers carried by the elementary excitations are the same as those of the doped holes. From the point of view of the renormalization group, the instability of the LLs found in reference [15] should result from a relevant operator generated by the t' term, which mediates an attraction between the charge and spin sectors. For example, the spin-charge recombination in the two-leg ladders arises from the superexchange interaction along the rung. The operators which mix the charge and spin sectors may occur in the six-fermion interactions in the continuum theory. But these are irrelevant operators in the weak coupling regime, i.e. $\Delta \ll 1$. Nevertheless, we cannot exclude the possibility that one of the six-fermion interactions mixing the charge and spin sectors becomes relevant in the strong regime, i.e. $\Delta = 1$. This problem is intimately related to the previous one: whether a phase transition exists between $\Delta \ll 1$ and $\Delta = 1$ or not. This issue deserves further study. In addition, to determine the exact value of $(t'/t)_c$ at $\Delta = 1$, further numerical work is warranted.

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Appendix A: Derivation of the continuum Hamiltonian

In this appendix, we provide the details for the derivation of the continuum Hamiltonian H_ψ from the lattice model given by equations (1) and (2). Equations (9) and (10) result from this procedure automatically.

Inserting equation (4) into equation (1) gives

$$H_h = H_0 + H_1 + H_2 + H_3 + H_4,$$

where

$$H_0 = -t \left(1 - \Delta \frac{n}{2}\right)^2 \sum_{j,\alpha} \left(c_{j+1\alpha}^\dagger c_{j\alpha} + \text{h.c.} \right) - t' \left(1 - \Delta \frac{n}{2}\right)^2 \sum_{j,\alpha} \left(c_{j+2\alpha}^\dagger c_{j\alpha} + \text{h.c.} \right), \quad (\text{A.1})$$

and

$$H_1 = t\Delta \left(1 - \Delta \frac{n}{2}\right) \sum_{j,\alpha} \left[c_{j+1\alpha}^\dagger c_{j\alpha} (:n_{j-\alpha} : + :n_{j+1-\alpha} : + \text{h.c.}) \right], \quad (\text{A.2})$$

$$H_2 = -t\Delta^2 \sum_{j,\alpha} \left(c_{j+1\alpha}^\dagger c_{j\alpha} :n_{j+1-\alpha} : :n_{j-\alpha} : + \text{h.c.} \right), \quad (\text{A.3})$$

$$H_3 = t'\Delta \left(1 - \Delta \frac{n}{2}\right) \sum_{j,\alpha} \left[c_{j+2\alpha}^\dagger c_{j\alpha} (:n_{j-\alpha} : + :n_{j+2-\alpha} : + \text{h.c.}) \right], \quad (\text{A.4})$$

$$H_4 = -t'\Delta^2 \sum_{j,\alpha} \left(c_{j+2\alpha}^\dagger c_{j\alpha} :n_{j+2-\alpha} : :n_{j-\alpha} : + \text{h.c.} \right). \quad (\text{A.5})$$

In the above, $:n_{j\alpha} := n_{j\alpha} - \frac{n}{2}$ and n is the electron density. In the following, we shall restrict to the case away from half-filling.

We first consider H_1 . Using equation (5), one may find the following operator product expansion (OPE) for $\alpha = \uparrow, \downarrow$

$$c_{j+1\alpha}^\dagger c_{j\alpha} / a_0 \approx \frac{\sin(k_F a_0)}{\pi a_0} + e^{ik_F a_0} J_{L\alpha} + e^{-ik_F a_0} J_{R\alpha} + \left(e^{2ik_F x} e^{ik_F a_0} \psi_{L\alpha}^\dagger \psi_{R\alpha} + \text{h.c.} \right). \quad (\text{A.6})$$

Furthermore, in terms of the ψ -fermions, the density operator for spin α , $:n_{j\alpha} :$, can be written as

$$:n_{j\alpha} : \approx a_0 \left[J_{L\alpha} + J_{R\alpha} + \left(e^{2ik_F x} \psi_{L\alpha}^\dagger \psi_{R\alpha} + \text{h.c.} \right) \right], \quad (\text{A.7})$$

where $J_{L(R)\alpha} = : \psi_{L(R)\alpha}^\dagger \psi_{L(R)\alpha} :$. With the help of equations (A.6) and (A.7) and neglecting the constant term, H_1 becomes

$$H_1 = 8ta_0\Delta \left(1 - \Delta \frac{n}{2}\right) \cos(k_F a_0) \int dx \left(J_{L\uparrow} J_{L\downarrow} + J_{R\uparrow} J_{R\downarrow} + J_L J_R - : \psi_{L\alpha}^\dagger \psi_{L\beta} \psi_{R\beta}^\dagger \psi_{R\alpha} : \right).$$

In terms of the identity

$$J_L^z J_L^z = \frac{1}{3} \mathbf{J}_L \cdot \mathbf{J}_L, \quad (\text{A.8})$$

we obtain

$$J_{L\uparrow} J_{L\downarrow} = \frac{1}{4} J_L J_L - J_L^z J_L^z = \frac{1}{4} J_L J_L - \frac{1}{3} \mathbf{J}_L \cdot \mathbf{J}_L. \quad (\text{A.9})$$

In addition, we have

$$: \psi_{L\alpha}^\dagger \psi_{L\beta} \psi_{R\beta}^\dagger \psi_{R\alpha} : = \frac{1}{2} J_L J_R + 2 \mathbf{J}_L \cdot \mathbf{J}_R. \quad (\text{A.10})$$

Inserting equations (A.9) and (A.10) into H_1 gives

$$H_1 = 2ta_0\Delta \left(1 - \Delta \frac{n}{2}\right) \cos\left(\frac{\pi n}{2}\right) \int dx \{ (:J_L J_L : + :J_R J_R : + 2J_L J_R) - \frac{4}{3} (: \mathbf{J}_L \cdot \mathbf{J}_L : + : \mathbf{J}_R \cdot \mathbf{J}_R : + 6 \mathbf{J}_L \cdot \mathbf{J}_R) \}. \quad (\text{A.11})$$

The same procedure is applied to H_3 except that t and a_0 are replaced by t' and $2a_0$, respectively. Accordingly, we have

$$H_3 = 4t'a_0\Delta \left(1 - \Delta \frac{n}{2}\right) \cos(\pi n) \int dx \{ (:J_L J_L : + :J_R J_R : + 2J_L J_R) - \frac{4}{3} (: \mathbf{J}_L \cdot \mathbf{J}_L : + : \mathbf{J}_R \cdot \mathbf{J}_R : + 6 \mathbf{J}_L \cdot \mathbf{J}_R) \}. \quad (\text{A.12})$$

Next we investigate H_2 . To proceed, we need the following OPE's:

$$J_{L\alpha}(z) \psi_{L\beta}(0) \sim -\frac{\delta_{\alpha\beta}}{2\pi z} \psi_{L\alpha}(0), \quad (\text{A.13})$$

$$J_{L\alpha}(z) \psi_{L\beta}^\dagger(0) \sim \frac{\delta_{\alpha\beta}}{2\pi z} \psi_{L\alpha}^\dagger(0).$$

Then, using equations (A.7) and (A.13), we obtain for $\alpha = \uparrow, \downarrow$

$$:n_{j+1\alpha} : :n_{j\alpha} : / a_0^2 \approx -\frac{2 \sin(k_F a_0)}{\pi a_0} \left\{ \frac{\sin(k_F a_0)}{2\pi a_0} + \cos(k_F a_0) (J_{L\alpha} + J_{R\alpha}) + \left(e^{2ik_F x} e^{ik_F a_0} \psi_{L\alpha}^\dagger \psi_{R\alpha} + \text{h.c.} \right) \right\} + : J_{L\alpha} J_{L\alpha} : + : J_{R\alpha} J_{R\alpha} : + 4 \sin^2(k_F a_0) J_{L\alpha} J_{R\alpha}. \quad (\text{A.14})$$

In equation (A.14), the $2nk_F$ terms with $n > 1$ are neglected. By using equations (A.6) and (A.14) and neglecting a constant term, H_2 becomes

$$H_2 = \frac{8ta_0\Delta^2 \sin(k_F a_0)}{\pi} \int dx \left\{ \cos^2(k_F a_0) (J_{L\uparrow} J_{L\downarrow} + J_{R\uparrow} J_{R\downarrow} + J_L J_R) - \frac{1}{4} (: J_{L\alpha} J_{L\alpha} : + : J_{R\alpha} J_{R\alpha} :) - : \psi_{L\alpha}^\dagger \psi_{L\beta} \psi_{R\beta}^\dagger \psi_{R\alpha} : \right\}.$$

With the help of equations (A.9), (A.10), and the identity we obtain

$$\begin{aligned} & : J_{L\alpha} J_{L\alpha} : + : J_{R\alpha} J_{R\alpha} := \\ & \frac{1}{2} (: J_L J_L : + : J_R J_R :) + \frac{2}{3} (: \mathbf{J}_L \cdot \mathbf{J}_L : + : \mathbf{J}_R \cdot \mathbf{J}_R :), \quad (\text{A.15}) \end{aligned}$$

we arrive at

$$\begin{aligned} H_2 = & \frac{t a_0 \Delta^2 \sin(\pi n / 2) \cos(\pi n)}{\pi} \int dx (: J_L J_L : + : J_R J_R : \\ & + 4 J_L J_R) - \frac{4 t a_0 \Delta^2 \sin(\pi n / 2)}{3 \pi} \int dx \{ [2 + \cos(\pi n)] \\ & \times (: \mathbf{J}_L \cdot \mathbf{J}_L : + : \mathbf{J}_R \cdot \mathbf{J}_R :) + 12 \mathbf{J}_L \cdot \mathbf{J}_R \}. \quad (\text{A.16}) \end{aligned}$$

With the same procedure, H_4 becomes

$$\begin{aligned} H_4 = & \frac{2 t' a_0 \Delta^2 \sin(\pi n) \cos(2 \pi n)}{\pi} \int dx (: J_L J_L : + : J_R J_R : \\ & + 4 J_L J_R) - \frac{8 t' a_0 \Delta^2 \sin(\pi n)}{3 \pi} \int dx \{ [2 + \cos(2 \pi n)] \\ & \times (: \mathbf{J}_L \cdot \mathbf{J}_L : + : \mathbf{J}_R \cdot \mathbf{J}_R :) + 12 \mathbf{J}_L \cdot \mathbf{J}_R \}. \quad (\text{A.17}) \end{aligned}$$

Finally, we turn into H_J . First, we have

$$\begin{aligned} & : n_{j+1} :: n_j : / a_0^2 \\ & \approx - \frac{2 \sin(k_F a_0)}{\pi a_0} \left[\frac{\sin(k_F a_0)}{\pi a_0} + \cos(k_F a_0) (J_L + J_R) \right] \\ & + : J_L J_L : + : J_R J_R : + 2 J_L J_R - 2 \cos(2 k_F a_0) \\ & \times : \psi_{L\alpha}^\dagger \psi_{L\beta} \psi_{R\beta}^\dagger \psi_{R\alpha} : \\ & \approx - \frac{2 \sin(k_F a_0)}{\pi a_0} \left[\frac{\sin(k_F a_0)}{\pi a_0} + \cos(k_F a_0) (J_L + J_R) \right] \\ & + : J_L J_L : + : J_R J_R : + [2 - \cos(2 k_F a_0)] J_L J_R \\ & - 4 \cos(2 k_F a_0) \mathbf{J}_L \cdot \mathbf{J}_R. \quad (\text{A.18}) \end{aligned}$$

Next, in terms of the ψ -fermions, the spin operator can be written as

$$\mathbf{S}_j \approx a_0 [\mathbf{J}_L + \mathbf{J}_R + (e^{2ik_F x} \mathbf{m} + \text{h.c.})], \quad (\text{A.19})$$

where $\mathbf{m} = \frac{1}{2} \psi_{L\alpha}^\dagger (\boldsymbol{\sigma})_{\alpha\beta} \psi_{R\beta}$. Using the OPE's

$$\begin{aligned} \mathbf{m}(x + a_0) \cdot \mathbf{m}^\dagger(x) &= \frac{3}{8 \pi a_0} \left[\frac{1}{\pi a_0} + i(J_L + J_R) \right] \\ &+ \frac{1}{2} \mathbf{J}_L \cdot \mathbf{J}_R - \frac{3}{8} J_L J_R, \quad (\text{A.20}) \\ \mathbf{m}^\dagger(x + a_0) \cdot \mathbf{m}(x) &= \frac{3}{8 \pi a_0} \left[\frac{1}{\pi a_0} - i(J_L + J_R) \right] \\ &+ \frac{1}{2} \mathbf{J}_L \cdot \mathbf{J}_R - \frac{3}{8} J_L J_R, \end{aligned}$$

$$\begin{aligned} & \mathbf{S}_{j+1} \cdot \mathbf{S}_j / a_0^2 \\ & \approx - \frac{3}{4 \pi^2 a_0^2} + : \mathbf{J}_L \cdot \mathbf{J}_L : + : \mathbf{J}_R \cdot \mathbf{J}_R : + 2 \mathbf{J}_L \cdot \mathbf{J}_R \\ & + e^{2ik_F a_0} \mathbf{m}(x + a_0) \cdot \mathbf{m}^\dagger(x) + e^{-2ik_F a_0} \mathbf{m}^\dagger(x + a_0) \cdot \mathbf{m}(x) \\ & \approx - \frac{3 \sin(k_F a_0)}{2 \pi a_0} \left[\frac{\sin(k_F a_0)}{\pi a_0} + \cos(k_F a_0) (J_L + J_R) \right] \\ & + : \mathbf{J}_L \cdot \mathbf{J}_L : + : \mathbf{J}_R \cdot \mathbf{J}_R : + [2 + \cos(2 k_F a_0)] \mathbf{J}_L \cdot \mathbf{J}_R \\ & - \frac{3}{4} \cos(2 k_F a_0) J_L J_R. \quad (\text{A.21}) \end{aligned}$$

Again, the $2nk_F$ terms with $n > 1$ in equations (A.18) and (A.21) are neglected. With the help of equations (A.18) and (A.21), we get

$$\begin{aligned} H_J = & J a_0 \int dx \left\{ : \mathbf{J}_L \cdot \mathbf{J}_L : + : \mathbf{J}_R \cdot \mathbf{J}_R : + 4 \cos^2\left(\frac{\pi}{2} n\right) \right. \\ & \times \mathbf{J}_L \cdot \mathbf{J}_R \left. \right\} - \frac{J a_0}{4} \int dx \left\{ : J_L J_L : + : J_R J_R : + 4 \cos^2\left(\frac{\pi}{2} n\right) \right. \\ & \left. \times J_L J_R \right\}. \quad (\text{A.22}) \end{aligned}$$

By collecting equations (A.11), (A.12), (A.16), (A.17), and (A.22), we obtain the continuum Hamiltonian H_ψ with the expressions (9) and (10).

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9. We also studied the spin-anisotropic $t - J$ model, i.e. the $t - J_{\perp} - J_z$ model, by using the approach of the deformed Hubbard operator. The continuum Hamiltonian we obtained is similar to H_{ψ} , except that the λ_s term in equation (7) becomes $\frac{\lambda_{\perp}}{2}(J_L^{\dagger} J_R^{-} + J_L^{-} J_R^{\dagger}) + \lambda_3 J_L^3 J_R^3$. Up to four-fermion interactions, we do not find the couplings between the charge and spin sectors. When $J_{\perp} \neq J_z$ ($\lambda_{\perp} \neq \lambda_3$), there are two phases: the Ising-Néel phase and the spin-Peierls phase. Both have a finite spin gap due to the breaking of the spin $SU(2)$ symmetry. We do not find the duality symmetry claimed in reference [7]
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